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Exam I MTH 111, Fall 2016

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$\frac{93}{95}$ Excellent

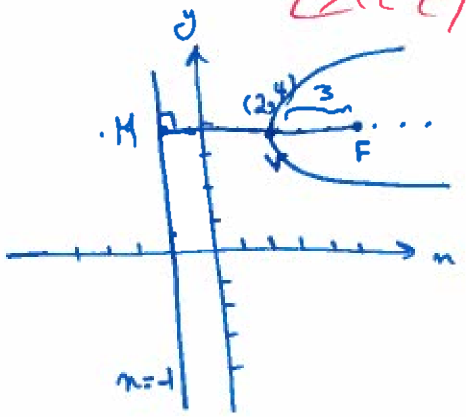
QUESTION 1. Given $12(x-2) = (y-4)^2$.

(i) Roughly, Sketch the graph of the given parabola.

$V = (2,4)$

$4d = 12 \rightarrow d = 3$

$M = (-1,4)$



(ii) What is the directrix line?

directrix: $\boxed{x = -1}$

(iii) What is the focus?

$\boxed{F = (5,4)}$

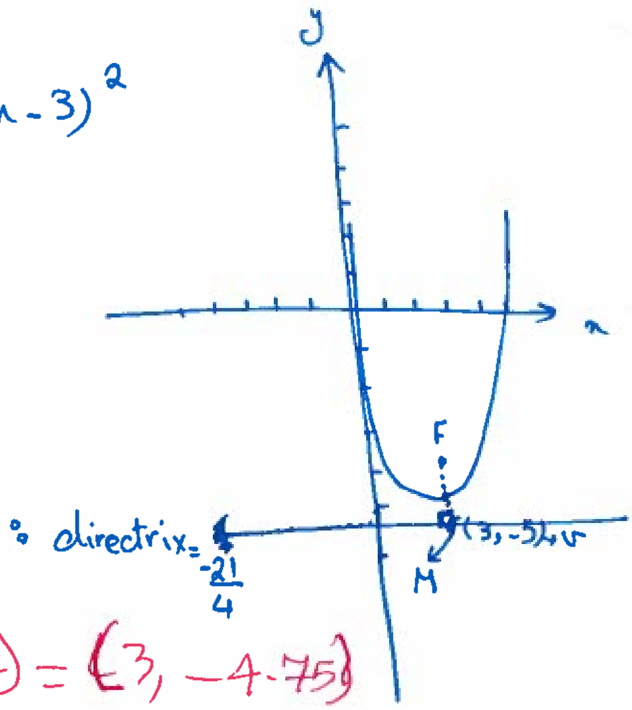
QUESTION 2. Given $y = x^2 - 6x + 4$

(i) Roughly, Sketch the graph of the given parabola.

$y - 4 = (x-3)^2 - 9 \rightarrow (y+5) = (x-3)^2$

$\boxed{V = (3,-5)}$

$\boxed{4d = 1 \rightarrow d = \frac{1}{4}}$



(ii) What is the directrix line?

$M = (3, -5 - \frac{1}{4})$

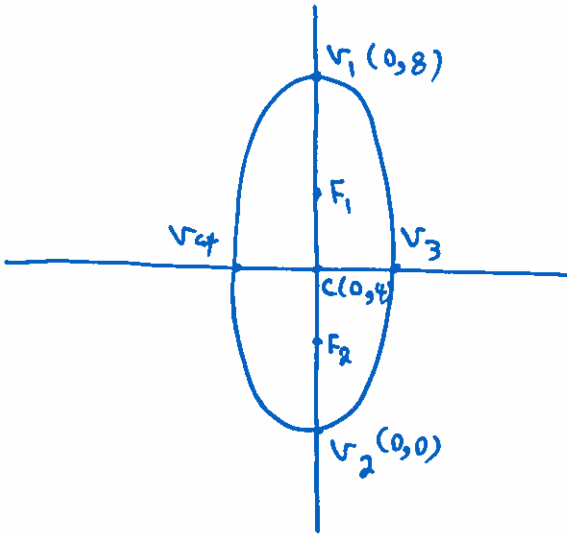
directrix: $y = -5 - \frac{1}{4} = \boxed{-5.25}$

(iii) What is the focus?

$\longrightarrow (3, -5 + \frac{1}{4}) = (3, -4.75)$

QUESTION 5. Given the ellipse $\frac{x^2}{7} + \frac{(y-4)^2}{16} = 1$.

(i) Roughly, Sketch the graph of the given ellipse.



$\boxed{C=(0,4)}$

$(\frac{K}{2})^2 = 16 \to \frac{K}{2} = 4 \to \boxed{K=8}$

$\boxed{b^2 = 7}$

(ii) Find the Ellipse-Constant K.

(iii) Find the two vertices of the major axis (the longer axis).

$|v_1 C| = \frac{K}{2} \to \boxed{v_1 = (0,8)}$

$|v_2 C| = K/2 \to \boxed{v_2 = (0,0)}$

$\boxed{\frac{(y-4)^2}{16} + \frac{(x-0)^2}{7} = 1}$

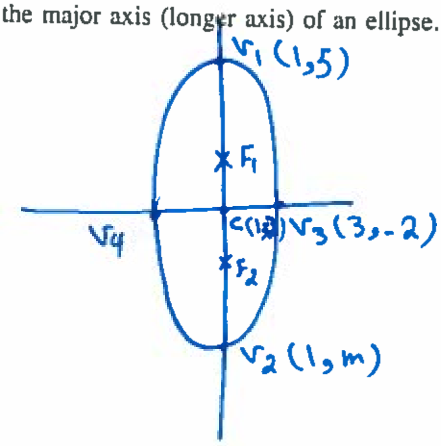
(iv) Find the two Foci: F_1, F_2.

$|CF_2| = |CF_1| = \sqrt{(\frac{K}{2})^2 - b^2} = \sqrt{16-7} = \sqrt{9} = 3$

$\boxed{F_1 = (0,7)}$ / $\boxed{F_2 = (0,1)}$

QUESTION 6. Given $V_1 = (1,5)$ and $V_2 = (1,m)$ are the vertices of the major axis (longer axis) of an ellipse. If $(3,-2)$ is another vertex of the ellipse, then

(i) Roughly, Sketch the graph of the given ellipse.



$\boxed{C=(1,-2)}$

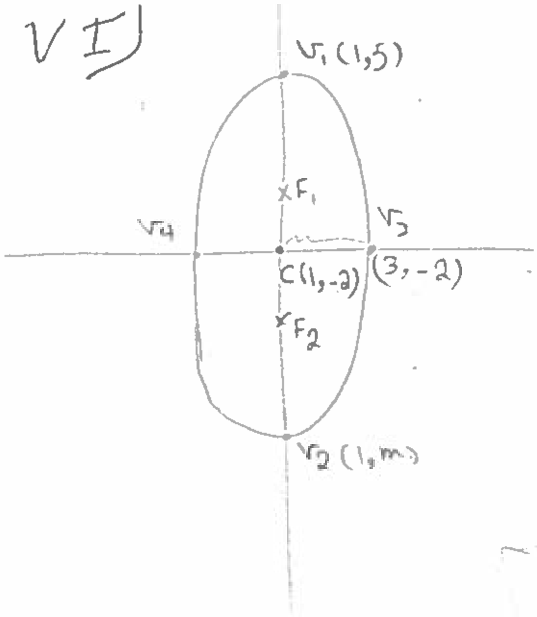
(ii) Find the Ellipse-Constant K.

$|Cv_1| = \frac{K}{2} = |5-(-2)| = 7 \to \boxed{K = 14}$

(iii) Find the value of m.

$v_2 = (1, -2-7) = \boxed{(1,-9)}$ m

VI)



$C=(1,-2)$

$K/2 = 7 \rightarrow \boxed{K=14}$

$\boxed{b=2}$

$\boxed{V_2=(1,-9)}$

$V_4=\boxed{(-1,-2)}$

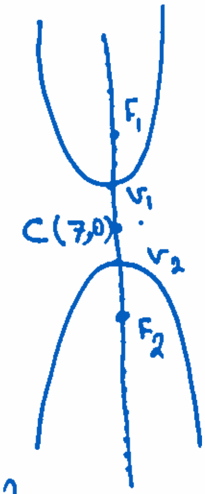
$CF_1 = \sqrt{49-4} = \sqrt{45} \Rightarrow$

$F_1 = (1, -2+\sqrt{45})$ $\boxed{F_2 = (1,-2-\sqrt{45})}$

$$\boxed{\frac{(y+2)}{49} + \frac{(x-1)^2}{4} = 1}$$

QUESTION 3. Given the hyperbola $\frac{y^2}{4} - \frac{(x-7)^2}{5} = 1$

(i) Roughly, Sketch the graph of the given hyperbola.



(ii) Find the two vertices, V_1 and V_2

$\left(\frac{K}{2}\right)^2 = 4 \rightarrow \frac{K}{2} = 2 \rightarrow \boxed{K = 4} \rightarrow |V_1V_2| \rightarrow |CV_1| = |CV_2| = 2$

$\boxed{V_1 = (7,2)}$ / $\boxed{V_2 = (7,-2)}$

(iii) Find the two Foci: F_1, F_2.

$|CF_1| = |CF_2| = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} = \sqrt{5+4} = \boxed{\sqrt{9} = 3}$

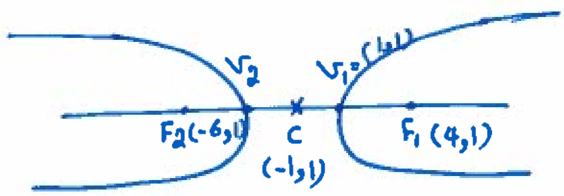
$\boxed{F_1 = (7,3)}$ / $\boxed{F_2 = (7,-3)}$

QUESTION 4. Given $F_1 = (4,1)$, $F_2 = (-6,1)$ are the foci of a hyperbola and $V_1 = (1,1)$ is one of the vertices.

(i) Find the hyperbola-constant K.

$C = \left(\frac{-6+4}{2}, 1\right) = (-1,1)$

$\frac{K}{2} = 2 \rightarrow \boxed{K = 4}$



(ii) Find the second vertex of the hyperbola.

$|CV_2| = K/2 \rightarrow \boxed{V_2 = (-3,1)}$

(iii) Find the equation of the hyperbola.

$|CF_1| = |CF_2| = 5 = \sqrt{\left(\frac{K}{2}\right)^2 + b^2} \rightarrow 5 = \sqrt{4 + b^2} \rightarrow 25 = 4 + b^2 \rightarrow \boxed{b^2 = 21}$

equation: $\boxed{\frac{(x+1)^2}{4} - \frac{(y-1)^2}{21} = 1}$

(iv) Find the fourth vertex of the ellipse.

$b = |CV_3| = 2 \rightsquigarrow \boxed{V_4 = (-1, -2)}$

(v) Find the two Foci: F_1, F_2 of the ellipse.

$|CF_1| = |CF_2| = \sqrt{(\frac{K}{2})^2 - b^2} = \sqrt{49 - 4} = \sqrt{45} \rightsquigarrow \boxed{F_1 = (1, -2 + \sqrt{45})}$ / $\boxed{F_2 = (1, -2 - \sqrt{45})}$

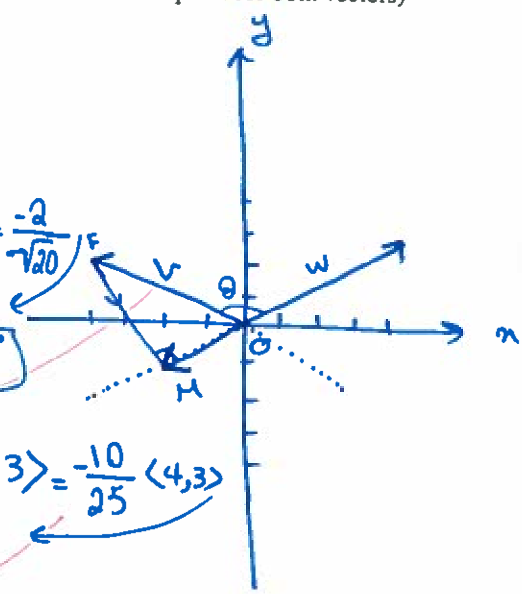
(vi) Find the equation of the ellipse.

$$\boxed{\frac{(y+2)^2}{49} + \frac{(x-1)^2}{4} = 1}$$

$\longrightarrow$ see back

QUESTION 7. Given $V = <-4, 2>$, $W = <4, 3>$ (you may consider (0, 0) as the initial point for both vectors)

(i) Sketch both vectors in the xy-plane



(ii) Find the angle between V, W (to the nearest 2 decimals)

$\cos\theta = \frac{V.W}{|V||W|} = \frac{-16+6}{(\sqrt{16+4})(\sqrt{16+9})} = \frac{-10}{5\sqrt{20}} = \frac{-2}{\sqrt{20}} = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5} \rightarrow \theta = \cos^{-1}\frac{-\sqrt{5}}{5} = \boxed{116.56°}$

(iii) Find $Proj_W^V$

$Proj_W^V = OM = \frac{W.V}{|W|^2} W = \frac{(-10)}{(\sqrt{16+9})^2} <4, 3> = \frac{-10}{25} <4, 3>$

$<\frac{-2 \times 4}{5}, \frac{-2 \times 3}{5}> = \boxed{<\frac{-8}{5}, \frac{-6}{5}>}$

(iv) Find $|Proj_W^V|$

$|Proj_W^V| = \frac{|W.V|}{|W|} = \frac{10}{\sqrt{16+9}} = \frac{10}{5} = \boxed{2}$

(v) Find Inj_W^V

$Inj_W^V = FM = V - Proj_W^V = <-4, 2> - <\frac{-8}{5}, \frac{-6}{5}> = <-4 + \frac{8}{5}; 2 + \frac{6}{5}>$

$<\frac{-12}{5}, \frac{16}{5}>$

$|Inj_W^V| = \sqrt{(\frac{-12}{5})^2 + (\frac{16}{5})^2}$

QUESTION 8. Find a parametric equations of the line that passes through the points $(1,3,1)$ and $(4,7,0)$.

$1 + a = 4 \to a = 3$
$3 + b = 7 \to b = 4$
$1 + c = 0 \to c = -1$

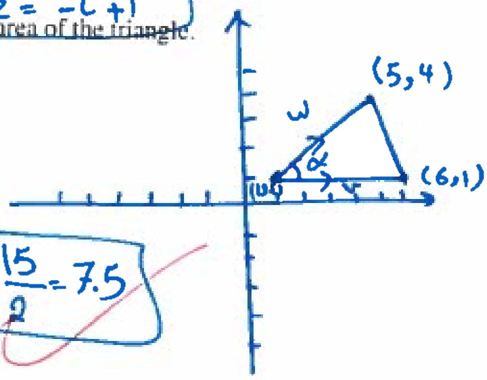
~~$x = 3t+1$~~ ~~$L: \langle 3t+1, 4t+3, -t+1\rangle$~~

$$\boxed{L: \begin{array}{l} x = 3t+1 \\ y = 4t+3 \\ z = -t+1 \end{array}}$$

QUESTION 9. Given $(1,1), (6,1), (5,4)$ are the vertices of a triangle. Find the area of the triangle.

$w = \langle 4,3\rangle$
$v = \langle 5,0\rangle$

$$\text{area} = \frac{|w \times v|}{2} = \begin{vmatrix} i & j & k \\ 4 & 3 & 0 \\ 5 & 0 & 0 \end{vmatrix}$$



$$\begin{vmatrix} 3 & 0 \\ 0 & 0 \end{vmatrix} i - \begin{vmatrix} 4 & 0 \\ 5 & 0 \end{vmatrix} j + \begin{vmatrix} 4 & 3 \\ 5 & 0 \end{vmatrix} k = -15k \to \boxed{\text{area} = \frac{15}{2} = 7.5}$$

QUESTION 10. Given that $P: 2x + 2y - z = 12$ is a plane in 3D.

(i) The point $Q = (1,1,12)$ is not in P. Find $|QP|$. $2x + 2y - z - 12 = 0$

$$|QP| = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}} = \frac{|2+2-12-12|}{\sqrt{4+4+1}} = \frac{|-20|}{\sqrt{9}} = \boxed{\frac{20}{3}}$$

(ii) Can we draw the vector $< 4, -5, -2 >$ inside the plane P? Why?

$N: \langle 2,2,-1\rangle$

$\langle 4,-5,-2\rangle \cdot \langle 2,2,-1\rangle = 8 - 10 + 2 = 0 \to$ yes we can because $V \cdot N = 0$

QUESTION 11. The plane $P_1: x + y + z = 4$ intersects the plane $P_2: -x + y + 4z = 6$ in a line L. Find a parametric equations of L.

$N_1 = \langle 1,1,1\rangle$

$N_2 = \langle -1,1,4\rangle$

$$N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ -1 & 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix} j + \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} k =$$

$$(4-1)i - (4+1)j + (1+1)k = \boxed{3i - 5j + 2k = L_N}$$

$z = 0 \to \begin{array}{l} x + y = 4 \\ -x + y = 6 \end{array} \Big\} \; 2y = 10 \to \boxed{y = 5}$ $\boxed{x = -1}$

$t\langle 3,-5,2\rangle + \langle -1,5,0\rangle =$

$$\boxed{\begin{array}{l} x = 3t - 1 \\ y = -5t + 5 \\ z = 2t \end{array}}$$

Faculty information

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